The role of near topography and building effects in vertical gravity gradients approximation

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Abstract
The gravitational effect of the topography and near-building structures and their contribution on the vertical gradient of gravity (VGG) was studied. The strong impact of near topography on the VGG values was found in the case of the mountainous areas – deviations of up to 88% of normal value were obtained by means of relative gravity measurements in selected parts of Slovakia. Newly developed software and a high-quality detailed digital terrain model of Slovakia was used for the evaluation of the topographical effect. The gravitational effect of near-building structures was estimated by means of simple 3D bodies approximation, i.e., rectangular or polygonal prisms. A very specific non-linear behaviour of VGG is demonstrated on model examples. A relatively good agreement between the measured and calculated (predicted) VGG values was achieved for a set of selected 32 real measurement points. The application of estimated (predicted) values of the VGG instead of the normal ones can lead to a quality improvement of global and local gravimetric reference networks, as well as prospecting VGG measurements.

Introduction
The requirement of precise determination of the vertical gradient of gravity (VGG) by means of relative gravity measurements is mainly connected to absolute gravity measurements and setting of global and local gravity reference networks. If we don’t know the actual value of the VGG, we can introduce into gravity value conversion between different height levels an error, which is usually considerably higher than the precision of the measurements itself. The determination of the VGG by means of field measurements is a time-consuming and uncomfortable procedure. This yields a need for the calculation of its approximated value. We have studied the possibility of estimating the actual VGG values on the basis of gravitational effect calculation of known surrounding mass bodies, namely the topography and/or man-made structures.

The significant effect of the near topography relief on the vertical gradient of gravity was recognized many years ago. Fajklewicz (1976) suggests ‘proper attention to even very small heights of the position of the lower measuring plate over the ground surface’ because of the strong and nonlinear effect of immediate relief.

Slovakia is a mountainous country, therefore we have carried out several VGG measurements by means of relative gravity meters in the rugged topography. We found that the near topography has the greatest impact on the anomalous values of the VGG. The anomalous values are considerably different from the normal gradient –0.3086⋅10⁻⁵ s⁻² (–0.3086 mGal/m or –3086 E). Up to this day we have acquired values of the VGG in the range from –0.580 up to –0.132⋅10⁻⁵ s⁻², which is a difference of 88% from the normal value. This differs significantly from the 30% given by LaFehr (1991). We have compared measured values of the VGG with calculated (predicted) values and we achieved relatively good agreement.

Naturally, the real values of the VGG depend on more factors, e.g., geological inhomogeneities. However, as we have proven, the near topography is the most important one, producing large anomalous values of the VGG. It is especially the case with absolute gravity points located in the underground cellars. The information about the surrounding area – the detailed digital terrain model (DTM) and building geometry give us the ability to estimate the actual value of the VGG to a relatively high level. This can lead to the improvement of gravity reference networks measurement. In the area of exploration utilization of VGG measurements, a precise removal of the effects of near topography improves the separation of the anomalous VGG components. Those are connected with searched anomalous subsurface objects as are cavities, geological structures and/or deposits objects, etc.

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Vertical gradient of gravity determination by means of relative gravity meters

We determine the VGG by relative gravity measurements using Scintrex CG-5 or CG-3M gravity meters on different height levels along the plumb line. The resolution of these gravity meters is $10^{-4}$ m s$^{-2}$ (1 μGal); standard accuracy achieved during good measurement conditions is on the level of several $10^{-4}$ m s$^{-2}$. In accordance with Fajklewicz (1976), we should use the term tower vertical gradient of gravity in order to distinguish it from the true point value of the VGG measured directly by gradiometers. We perform the measurements at a minimum of two and a maximum of four height levels in the case of the absolute gravity points. The measurement range is approximately 0.25-1.5 m above the ground (Figure 1). We suppose these height intervals can be maintainable at such lower values thanks to relatively high-gravity data acquisition accuracy of modern instruments. There are some practical reasons why we keep such height intervals. Many of absolute gravity points are situated inside small underground rooms, so there is no possibility to perform the measurements on the higher levels. Moreover, our task is to define the VGG on approximately the same height interval as is the measurement level of absolute gravity meters to transform properly the measured gravity to the geodetic benchmark. That is about 1.3 m above the ground in the case of FG-5 absolute gravity meter.

We can determine the VGG simply as a constant value by dividing $\Delta g / \Delta h$ in the case of two measured levels. We can also use a linear or polynomial approximation of the $g(h)$ function and evaluate VGG by derivative $\partial g / \partial h$ in the case of more height levels. The $g$ is the drift-corrected relative value of the gravity acceleration and $h$ is the height of the gravity meter sensor above the ground.

Near topography and building effects calculation

We have calculated the topography effect and its contribution to the VGG using newly developed software Toposk (Marušiak and Mikuška 2012) in the frame of our current project. The calculation is realized by summation of grav-
We can calculate also the gravitational effect of near-building structures – underground cellars, walls, concrete pillars below the measurement point (see Figure 1). We used a modelling software Potent v.4.11.06 (Potent 2010), which enables the calculation of the gravitational effect of simple 3D bodies, such as polygonal prisms etc. In Figure 2 we demonstrate the effect of a concrete pillar with difference density $+500 \text{ kg/m}^3$ and the effect of surrounding walls with the density $+2350 \text{ kg/m}^3$. The dimensions of the assumed room are $2 \times 2 \times 2.5 \text{ m}$. The absolute gravity points are usually situated in such conditions (Figure 1). As we can see, these near-building structures have a non-negligible contribution to the VGG. These effects are positive as well as negative. In addition they are nonlinear.

As we have already mentioned, the common practice is to situate the absolute gravity points inside the underground cellars. This gives rise to a special anomalous and nonlinear behaviour of the VGG also in the areas with flat surrounding topography. We evaluate this effect in two steps. First, we calculate the effect of the topography without ‘empty’ space around the measurement point. Then we calculate the effect of ‘missing’ masses with the same density as for the topography, but with the negative sign. We present two common situations in Figure 3. We demonstrate separately gravitational effects of 3D polyhedral bodies (Pohánka 1988) and spherical layer segments (Mikuška et al. 2006). We calculated the gravitational effect of topography masses up to the distance of 166.7 km (Hayford-Bowie zone O₂), which is the standard used for the terrain corrections in gravimetry. The effect of the topography beyond the Hayford-Bowie zone L (28,800 m) is very small and almost identical for all height levels. The contribution of distant zones beyond the 166.7 km to the VGG values is almost negligible (Mikuška et al. 2006). For our current calculations we have used the latest version of detailed high quality DTM of Slovakia DMR-3 (DTM3 2012). This model was created by digitizing topographic maps in the scale 1: 10 000 and 1: 25 000, coordinate system S-1942, resolution 10 x 10 m. The control test of approx. 2000 points of the national spatial network showed mean accuracy of 3.7 m. The expected accuracy in the forest areas is about 10 m. We used SRTM model (Jarvis et al. 2008) for the calculation of farther zones beyond the boundaries of Slovakia. It is of course necessary to keep the actual position of the gravity sensor during measurements in regard to the surrounding topography, especially in the case of underground points. Therefore, we adapt true heights of the measured points to the heights interpolated from DTM during the calculation within the near zone up to 250 m.

We can calculate also the gravitational effect of near-building structures – underground cellars, walls, concrete pillars below the measurement point (see Figure 1). We used a modelling software Potent v.4.11.06 (Potent 2010), which enables the calculation of the gravitational effect of simple 3D bodies, such as polygonal prisms etc. In Figure 2 we demonstrate the effect of a concrete pillar with difference density $+500 \text{ kg/m}^3$ and the effect of surrounding walls with the density $+2350 \text{ kg/m}^3$. The dimensions of the assumed room are $2 \times 2 \times 2.5 \text{ m}$. The absolute gravity points are usually situated in such conditions (Figure 1). As we can see, these near-building structures have a non-negligible contribution to the VGG. These effects are positive as well as negative. In addition they are nonlinear.
the VGG contribution of the topography (black dashed lines) and the contributions of the cellars (black solid lines), as well as their summary contribution (red lines). The topography in these models is represented by the truncated spherical layer up to 166.7 km with constant thickness. In the first case (left-hand picture in Figure 3), the measured height range crosses the boundary of surrounding topography because we have assumed the depth of the cellar 1.2 m with regard to surrounding relief. While the VGG contributions of the topography and cellar show large discontinuities in this case, total VGG is considerably nonlinear but without any discontinuity. It is because of relatively large cellar dimensions. We would get a real measurable discontinuity only in the case of very narrow underground objects (e.g. boreholes).

The second model (right-hand picture in Figure 3) represents the situation with a deeper cellar 3 m below the relief. We see that inside such small cellars we can expect strong anomalous VGG values. It is important to keep real positions of the calculation points with regard to the cellar geometry as well as near topography.

Figure 4: The elevation map of Slovakia with marked points of VGG measurements. The black dots represent absolute gravity points of the Slovak gravimetric reference network, red ones represent field points. Red arrows indicate the points with opposite extreme measured values of the VGG (see also Figure 5 and 6).

Figure 5: The lowest VGG value $-0.580 \times 10^{-5} \text{ s}^{-2}$ was acquired on the top of the second highest peak of Slovakia, Lomnický štít, 2631 m.
In all mentioned cases, it is very important to define real geometry and object density. The uncertainty of used densities can yield significant errors in the calculation of these important effects.

Real data study

We present the results of 32 measurements of the VGG (by means of relative gravity meters CG-5 and CG-3M) from different parts of Slovakia (Figure 4). We have measured vertical gradients on absolute gravity points of the national gravimetric network. This has been done with the aim of transforming the measured absolute gravity from the height of the measurement to the geodetic benchmark. Repeated VGG measurements on selected absolute points were realized during the past few years. Our standard procedure consists of two measurement cycles on four height levels with 3-5 readings on each level.

Besides the measurements on absolute gravity points, we have performed several VGG measurements in the field conditions. The goal was to find the most anomalous VGG values on the basis of former calculations of the topography effect. The ‘anomalous’ means the more deviated from the normal value $-0.3086 \times 10^{-5}$ s$^{-2}$. Actual results are shown in Figure 7. Measured values of the VGG are processed as constant values (black symbols). They are shown together with the values corrected for the topography effect (red ones). The measurements on the points situated in the underground cellars were corrected also for the gravita-
Absolute gravity points are grouped in the right-hand side of the picture; they are marked by triangles. They have smaller amplitudes, because these points are not situated on the places with extreme local topography. The larger amplitudes of the last five points belong to underground absolute gravity points. Points measured in the field conditions are in the left-hand side of the graph (crosses). We have chosen these points because of the rugged topography. For example, the point with extreme negative value of the VGG $-0.580 \times 10^{-5}$ s$^{-2}$ was measured on the second-highest peak in Slovakia, 2631 m above sea level (see Figure 5).

It is interesting to see how the corrected values are closer to the normal VGG value. The measurements were not corrected for the effect of the pillars (walls) because we do not have reliable information about their dimensions or densities. The topography effect was calculated in most cases for the density 2670 kg/m$^3$. We have recognized that this density leads to over-corrections in the case of most underground points because the cellars are made in the environment with evidently lower density. Therefore, we have used the density 2200 kg/m$^3$ in such cases.

Figure 8: The comparison of anomalous VGG values on the set of 32 measurements (see Figure 7) expressed as percental values of the normal VGG ($-0.3086 \times 10^{-5}$ s$^{-2}$). Left-hand graph represents measured VGG values, right-hand graph represents VGG corrected for topography and building effects.

Figure 9: The estimated values of the VGG calculated as the sum of VGG contributions of the topography and the normal value $-0.3086 \times 10^{-5}$ s$^{-2}$ (upper part). Two local areas selected around the points with extreme measured VGG values (green crosses, see also Figure 4) are presented. The red and blue triangle mark the extreme calculated values. The calculation was made in the grid 50 x 50 m using the digital terrain model DMR-3 (lower part).
In Figure 8 are shown the histograms of measured as well as corrected VGG values expressed as percental values of the normal VGG. Both pictures (Figure 7 and 8) demonstrate the significance of the near topography concerning the vertical gradients. It is evident that the corrections for topography play the most important role at the approximation of the anomalous VGG values.

Besides the particular measurement points, we have made a calculation for the whole area of Slovakia on points extrapolated from DMR-3 to get an idea about the expected values of the VGG. We have estimated VGG contribution of the topography simply as a difference of the calculated values of the topography effect on two height levels: 1.25 m and 0.25 m above the ground. The normal VGG value $-0.3086 \cdot 10^{-5}$ s$^{-2}$ was then added to the calculated value. We present the results of the calculation over two local areas in Figure 9. In general, negative deviations from normal VGG values are inherent to the points situated on the local elevations (peaks, ridges), positive deviations are typical for local depressions (valleys). This is in accordance with measured results. The estimated VGG values are in the range from $-0.570$ to $-0.075 \cdot 10^{-5}$ s$^{-2}$. While the reality of the minimum value of the VGG was proved by field measurements, more interesting is the upper limit $-0.075 \cdot 10^{-5}$ s$^{-2}$ (red triangle in Figure 9), which is not yet confirmed by our field measurements. The estimations depend on the quality and resolution of used DTM, especially in the case of narrow valleys. We have compared these estimations at local areas with the results obtained using more detailed available DTMs. These models with resolution 1 x 1 m are derived from photogrametry (DTM4 2013, personal communication) or LIDAR data (Hofierka and Čebecauer 2007; Hofierka et al. 2013). We achieved different results from those using DMR-3, generally with smaller amplitudes of the anomalous VGG values.

The estimations depend on the quality and resolution of the topography and buildings corrections have been applied. In the case of the measured gravity value transformation this represents an improvement of mean gravity error approx. from 0.086 to 0.015 $\cdot 10^{-7}$ m s$^{-2}$ per one metre of height difference. The application of estimated (predicted) values of the VGG instead of the normal one in cases of unknown actual values can lead to a quality improvement of global and local gravimetric reference networks, as well as prospecting VGG measurements.

Conclusions

Determined values of vertical gradient of gravity can strongly differ from its normal value $-0.3086 \cdot 10^{-5}$ s$^{-2}$ in mountainous countries like Slovakia (up to 88%). We have measured the VGG values using relative gravity measurements in the range from $-0.580 \cdot 10^{-5}$ s$^{-2}$ (Lomnický štít, 2631 m) up to $-0.132 \cdot 10^{-5}$ s$^{-2}$ (Zádielská tiesňava, 427 m). We have found relatively good agreement between measured values of VGG and their estimated or predicted values. We have calculated gravitational effects of the topography by means of newly developed software using a detailed digital terrain model which contributes to the VGG values. We also calculated the gravitational effect of near-building structures by means of the software Potent to consider its impact on the VGG. The simple models proved the nonlinearity of the VGG, especially in the case of underground absolute gravity points. The set of 32 measured points showed the decrease of mean anomalous VGG values from 28 to 5% of the normal value after the topography and buildings corrections have been applied. In the case of the measured gravity value transformation this represents an improvement of mean gravity error approx. from 0.086 to 0.015 $\cdot 10^{-7}$ m s$^{-2}$ per one metre of height difference. Acknowledgements

The authors are grateful to the Slovak Research and Development Agency APVV, grants No. APVV-0194-10, APVV-0724-11 and the Slovak Grant Agency VEGA, grants No. 2/0067/12, 1/0095/12 for their support. This paper is also the result of the implementation of the project: the National Centre of Earth's Surface Deformation Diagnostic in the area of Slovakia, ITMS 26220220108 supported by the Research and Development Operational Programme funded by the ERDF. We would like to express our thanks to J. Mikuška and I. Marušiak from G-trend Ltd. for the permission to use their software Toposk in the frame of this published study.

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